Evaluate $\int \sec^6 x \tan^6 x \, dx$. $= \int \sec^4 x \tan^6 x \cdot \sec^2 x \, dx$

SCORE: _____ / 4 PTS

$u = \tan x$ (1)

$du = \sec^2 x \, dx$

$= \int (u^2+1)^2 u^6 \, du$ (1)

$= \int (u^{10} + 2u^8 + u^6) \, du$ ($\frac{1}{2}$)

$= \frac{1}{11}u^{11} + \frac{2}{9}u^9 + \frac{1}{7}u^7 + C$ ($\frac{1}{2}$)

$= \frac{1}{11}\tan^{11} x + \frac{2}{9}\tan^9 x + \frac{1}{7}\tan^7 x + C$ (1)

SUBTRACT ($\frac{1}{2}$) IF YOU FORGOT "+C"

Prove the reduction formula $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$ (where $n \neq 0$). **SCORE: _____ / 6 PTS**

<u>NOTE: You must show how to get this formula.</u>

<u>You will receive 0 credit if your "proof" is differentiating both sides of the equation.</u>

$$\begin{array}{ccc} \sin^{n-1} u & + & \sin u \\ (n-1)\sin^{n-2} u \cos u & - & -\cos u \end{array} \quad 1$$

$$\int \sin^n u\,du = -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u \cos^2 u\,du \quad 1\tfrac{1}{2}$$

$$= -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u (1-\sin^2 u)\,du$$

$$1 \quad = -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u\,du - (n-1)\int \sin^n u\,du \quad 2$$

$$n\int \sin^n u\,du = -\sin^{n-1} u \cos u + (n-1)\int \sin^{n-2} u\,du$$

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du \quad \tfrac{1}{2}$$

Evaluate $\int (2-x-3x^2)\cos\frac{x}{2}\,dx$. $= 2(2-x-3x^2)\sin\frac{x}{2}$ $\frac{1}{2}$

SCORE: _____ / 4 PTS

$+4(-1-6x)\cos\frac{x}{2}$ $1\frac{1}{2}$

$+48\sin\frac{x}{2}+C$ 1 $= \boxed{(52-2x-6x^2)\sin\frac{x}{2} - (4+24x)\cos\frac{x}{2}} + C$ 1

$2-x-3x^2$	$+$	$\cos\frac{x}{2}$
$-1-6x$	$-$	$2\sin\frac{x}{2}$
-6	$+$	$-4\cos\frac{x}{2}$
0		$-8\sin\frac{x}{2}$

SUBTRACT $\frac{1}{2}$ IF YOU FORGOT "+C"

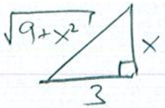
Evaluate $\int (9+x^2)^{\frac{3}{2}}\, dx$.

SCORE: _____ / 7 PTS

$9+x^2 = 9\left(1+\frac{1}{9}x^2\right) = 9\left(1+\tan^2\theta\right) = 9\sec^2\theta$

$\frac{1}{9}x^2 = \tan^2\theta$

$\frac{x}{3} = \tan\theta$



1 $x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\frac{1}{2}$ 1

$= \int (3\sec\theta)^3 \cdot 3\sec^2\theta\, d\theta$

$\frac{1}{2}$ $= 81\int \sec^5\theta\, d\theta$ $\frac{1}{2}$

$= 81\left(\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int \sec^3\theta\, d\theta\right)$ 1

1

$= \frac{81}{4}\sec^3\theta\tan\theta + \frac{243}{4}\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|\right) + C$

$= \frac{81}{4}\sec^3\theta\tan\theta + \frac{243}{8}\sec\theta\tan\theta + \frac{243}{8}\ln|\sec\theta + \tan\theta| + C$

$= \frac{81}{4}\left(\frac{\sqrt{9+x^2}}{3}\right)^3 \frac{x}{3} + \frac{243}{8}\,\frac{\sqrt{9+x^2}}{3}\,\frac{x}{3} + \frac{243}{8}\ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C$

1

$= \frac{1}{4}x\sqrt{9+x^2} + \frac{27}{8}x\sqrt{9+x^2} + \frac{243}{8}\ln\left|\sqrt{9+x^2} + x\right| + C$

$\frac{1}{2}$

SUBTRACT $\frac{1}{2}$ IF YOU FORGOT "+C"

Evaluate $\int e^{-6x} \cos 2x \, dx$. $= -\frac{1}{6}e^{-6x}\cos 2x + \frac{1}{18}e^{-6x}\sin 2x$ (1) $- \frac{1}{9}\int e^{-6x}\cos 2x\, dx$ $(1\frac{1}{2})$ **SCORE: ____ / 5 PTS**

$(\frac{1}{2})$

$$\begin{array}{ll} \cos 2x & e^{-6x} \\ -2\sin 2x & -\frac{1}{6}e^{-6x} \\ -4\cos 2x & \frac{1}{36}e^{-6x} \end{array}$$

+ −

±

(1) $\frac{10}{9}\int e^{-6x}\cos 2x\, dx = -\frac{1}{6}e^{-6x}\cos 2x + \frac{1}{18}e^{-6x}\sin 2x$

$\int e^{-6x}\cos 2x\, dx = -\frac{3}{20}e^{-6x}\cos 2x + \frac{1}{20}e^{-6x}\sin 2x + C$

(1)

SEE ALTERNATE SOLUTION BELOW

BUT GRADE AGAINST ONE VERSION ONLY

SUBTRACT $(\frac{1}{2})$

IF YOU FORGOT "+C"

OR

Evaluate $\int e^{-6x} \cos 2x \, dx$. $= \frac{1}{2} e^{-6x} \sin 2x - \frac{3}{2} e^{-6x} \cos 2x$ 1

$\frac{1}{2}$ $- 9 \int e^{-6x} \cos 2x \, dx$ $1\frac{1}{2}$

SCORE: _____ / 5 PTS

$$\begin{array}{ccc} e^{-6x} & + & \cos 2x \\ -6e^{-6x} & - & \frac{1}{2}\sin 2x \\ 36e^{-6x} & \pm & -\frac{1}{4}\cos 2x \end{array}$$

1 $10 \int e^{-6x} \cos 2x \, dx = \frac{1}{2} e^{-6x} \sin 2x - \frac{3}{2} e^{-6x} \cos 2x$

1 $\int e^{-6x} \cos 2x \, dx = \frac{1}{20} e^{-6x} \sin 2x - \frac{3}{20} e^{-6x} \cos 2x + C$

SUBTRACT $\frac{1}{2}$
IF YOU FORGOT "+C"

Evaluate $\int \frac{(\ln x)^2}{\sqrt{x}}\,dx$. $= 2x^{\frac{1}{2}}(\ln x)^2 - 8x^{\frac{1}{2}}\ln x + 16x^{\frac{1}{2}} + C$ SCORE: _____ / 4 PTS

$\underline{2x^{\frac{1}{2}}(\ln x)^2}$ (1) $\underline{-8x^{\frac{1}{2}}\ln x}$ (2) $\underline{+16x^{\frac{1}{2}}}$ (1)

SUBTRACT $\frac{1}{2}$ IF YOU FORGOT "+C"

$(\ln x)^2 \quad + \quad x^{-\frac{1}{2}}$

$\frac{2\ln x}{x} \quad 2x^{\frac{1}{2}}$

$* x$ - - - - - - - $* \frac{1}{x}$

$2\ln x \quad - \quad 2x^{-\frac{1}{2}}$

$\frac{2}{x} \quad 4x^{\frac{1}{2}}$

$* x$ - - - - - - - $* \frac{1}{x}$

$2 \quad + \quad 4x^{-\frac{1}{2}}$

$0 \quad 8x^{\frac{1}{2}}$